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# Weak semiring structure of $\beta$ -languages

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**Abstract.**  $\beta$ -languages of order  $n$  have been introduced by the authors of [6] and their equivalence with the semi-deterministic push-down automaton (SDPDA) languages of order  $n$  has been established in [6, 7]. In this paper, we make a study of various closure properties of  $\beta$ -languages. We show that the class of  $\beta$ -languages of order  $n$  forms a weak semiring under union ( $n \geq 2$ ) and concatenation ( $n \geq 1$ ).

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